ON HOTELLING'S COMPETITION WITH GENERAL PURPOSE PRODUCTS[1]

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ABSTRACT. This paper extends the traditional Hotelling's model of spatial competition by allowing firms to choose the degree of general purposeness of their products before they compete in prices. The degree of general purposeness is approximated by endogenizing the per–unit transportation cost coefficients. The game presents a continuum of perfect Nash equilibria featuring no price competition. In equilibrium, firms behave as 'specialist' by choosing high transportation cost coefficients. This allows them to extract all the marginal consumer's rent and to perfectly segment the market. Moreover, market is entirely served by both firms regardless the value of the consumer's reservation price.

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1. Introduction

Since the seminal contribution made by Hotelling (1929), the analysis of spatial competition models has represented a core issue in the field of Industrial Organization. Using a simple duopoly model with differentiated products, Hotelling showed that too similar product configuration would arise in equilibrium, result that became known as the principle of minimum differentiation. Notwithstanding, fifty years later D'Aspremont et al. (1979) pointed a flat in the Hotelling's analysis and proved that the principle of minimum differentiation is not robust to the choice of the distance function. Since then, models with nonlinear transportation cost functions [D'Aspremont et al. (1979); Stahl (1982)], alternative specifications for the demand function [Economides (1986; 1984); Anderson (1997); Puu (2002)], different distribution functions for the population of consumers [Neven

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(1986)], or a mix thereof have been proposed[1]. Yet all these models approach the problem of product differentiation through the analysis of the choice of location made by two suppliers in a geographic market. Furthermore, all the aforementioned models assume that transportation (or disutility) costs are function of consumer tastes alone and therefore take them as exogenous in firm's maximization decisions. Nevertheless, some important aspects of the theory of product differentiation can be captured by relaxing this last assumption. One way to accomplish this is by bringing out general purpose products into the model. The idea of general purpose products dates back to von Ugern-Stemberg (1988) who exemplifies it through the IBM 360 series computers. According to von Ugern-Stemberg, the characteristic property of a general purpose computer is that it can perform the functions of a variety of specific purpose computers almost equally well. This general purposeness is not reached by choosing a particular price–location pair but by affecting the transportation cost that consumers need to bear in a monopolistic model such as the Hotelling's (1929) one. Indeed, von Ugern-Stemberg (1988) suggests to model this type of products by endogenizing transportation costs in the standard circular road model of Salop (1979). Later on, Hendel and Neiva de Figueiredo (1997) proposed a variation of the von Ugern-Stemberg (1988) model which differs from the former in the timing of the game. Thus, while von Ugern-Stemberg (1988) proposes a model in which the focus and price competition take place simultaneously, Hendel and Neiva de Figueiredo (1997) suggest a sequential game in which focus competition occurs prior price competition. Both models analyze the symmetrical perfect equilibria.

Endogenization of transportation cost coefficients in spatial competition models *à la Hotelling* can also be viewed as a mechanism to account for all those costs generated by the bilateral relationship between buyer and seller that are under the firm's control. In this context, transportation cost as a measure of the consumer's disutility should reflect all the costs associated with the purchase process and not only the logistic ones, wherein transportation costs are included. Alternatively, it appears quite unrealistic to assume that firms will take as granted a variable that can be used to soften price competition. When we fix the product characteristics (by fixing firms' locations), transportation cost coefficients become the unique source of differentiation available for firms. This is so because for given locations, a higher transportation cost reduces the firm's incentives to compete aggressively in prices by making the neighboring clientele become more captive. This in turn increases the 'monopoly power' enjoyed by each firm. Therefore, endogenous transportation cost coefficients

[1]For extensive surveys on models of spatial competition see Philips and Thisse (1982) and Kilkenny and Thisse (1999).

appear as an attractive approach to model those aspects of product competition in differentiated markets that have not been taken into consideration by traditional models of spatial competition.

In the present paper we analyze a spatial competition model that differs from previous models with endogenous transportation costs in two respects. First, we study a modified version of the linear model of Hotelling (1929) instead of the circular road model of Salop (1979). Second, we analyze not only symmetrical perfect equilibria but also characterize asymmetrical equilibria. For simplicity, the length of the city is assumed to be equal to one. The game takes place in two stages. In the first stage firms select non cooperatively the degree of purposeness (or focus) of their products which is approximated by the choice of a transportation cost coefficient. In the second stage, firms compete in prices. The extend to which products are differentiated in the characteristic space is assumed exogenous. More precisely, firms are assumed to be located at both ends of the linear city. Thus, maximal differentiation in the characteristic product space is imposed exogenously into the model. This assumption is made with the aim of isolating the effects of endogenous transportation costs over price competition in spatially differentiated markets. Under the argument that, for fixed locations, a higher transportation cost reduces the incentives to compete aggressively in prices, the assumption of maximal horizontal differentiation seems harmless if the focus is paid on the consequences of the selection of transportation cost coefficients over price competition. After all, if firms were not fully differentiated in the product characteristic space, they would face even greater incentives if they wished to differentiate through transportation costs[2].

The rest of the paper is organized as follows. The model set–up is the topic of section 2, whereas the pricing subgame is the focus of section 3. Section 4 characterizes the perfect Nash equilibria for our two–stage model of spatial competition. Finally, Section 5 ends the paper with some comments and concluding remarks.

2. Model set–up

The model we study is a variant of the Hotelling's spatial duopoly model. Consider a two–stage game, denoted by Γ, with two firms and a continuum of consumers. These consumers are distributed on a linear city of unit length according to a uniform density function. Each consumer is entitled to buy at most one unit of the commodity. In order to obtain the good, each consumer must bear a transportation cost $t_i x$, where x is the distance between the location of the consumer and store

[2]Our argument runs over the same line as the one given by Boccard and Wauthy (2000).

i, and t_i is the transportation cost coefficient. We assume, without loss of generality, $t_i \in [\underline{t}, \overline{t}]$. Firm i and j are located at the two ends of the city and these locations are exogenously determined into the model. Consequently, if x is the distance between the location of the consumer and store i,$(1-x)$ is her distance to store j.

The game takes place in two stages. In the first stage, firms simultaneously and independently select transportation cost coefficients, $(t_i, t_j) \in [\underline{t}, \overline{t}]^2$. In the second stage, firms observe t's and compete non cooperatively in prices. In this stage, firm i's pricing strategy, denoted by p_i, is a function that maps any pair of transportation cost coefficients (t_i, t_j) onto the interval $[0, +\infty]$. The production cost is assumed to be identical and equal to c per unit for both firms. Consumers have a common and finite reservation price equal to $\bar{s}$, $\bar{s} > c$. Production costs and consumer's reservation price are common knowledge. We are looking for Subgame Perfect Nash Equilibrium (SPNE) of the two–stage game. Consequently, we proceed to solve the game by backward induction, considering initially any possible history (t_i, t_j) for the first-stage of the game.

3. The pricing subgame

Let (p_i, p_j) be the prices charged by firms i and j respectively. Recalling that x represents the consumer's distance to firm i and $(1-x)$ her distance to firm j, the consumer's utility function is given by,

$$U(p_i, p_j) = \begin{cases} \bar{s} - p_i - t_i x & \text{if she buys from firm } i; \\ \bar{s} - p_j - t_j(1-x) & \text{if she buys from firm } j; \\ 0 & \text{otherwise.} \end{cases}$$

If prices and transportation cost coefficients are not too high, firm i faces a demand that is equal to the number of consumers who find cheaper to buy from this firm. This demand is the solution to $\bar{s} - p_i - t_i x^* = \bar{s} - p_j - t_j(1-x^*)$ and represents the location of the consumer who is indifferent from buying at either firm. If prices or transportation cost coefficients are too high, not all consumers will accept to buy the good and the firms demand will be given by x^m, where x^m solves $\bar{s} - p_i - t_i x^m = 0$. Finally, considering that one's demand can not exceed 1 nor be lower than 0, we can write the demand function as follows:

$$D_i(p_i, p_j, t_i, t_j) = \max\left[\min\left\{\frac{p_j - p_i + t_j}{t_i + t_j}; \frac{\bar{s} - p_i}{t_i}; 1\right\}; 0\right]$$

A diagrammatic representation of the demand function along with a graphical analysis of its derivation can be found in Appendix A. Using this demand function, we can write firm i's best-response correspondence as follows:

$$B_i(p_j, t_i, t_j) = \arg\max_{p_i} \left[(p_i - c) \max \left[\min \left\{ \frac{p_j - p_i + t_j}{t_i + t_j}; \frac{\bar{s} - p_i}{t_i}; 1 \right\}; 0 \right] \right]$$

Notice that both the demand and profit functions are not continuously differentiable everywhere. This poses some analytical difficulties but conceptually the problem is straightforward. Because of the kinked structure of the demand function, the best–response correspondence is made by three segments. Thus, depending on where the best-response correspondences cross on the p_i and p_j space, different types of equilibria can arise. The first segment of the best–response correspondence is upward slopping, i.e., prices are strategic complements. When the best–response correspondences cross in this range, the equilibrium corresponds to the typical Hotelling equilibrium; the entire market is covered and the marginal consumer (the one just indifferent from buying at either firm) obtains positive rents in equilibrium. This case occurs when the transportation cost coefficients, t_i and t_j, are low relative to $\bar{s}$. The second section of best–response correspondence can be either upward or downward slopping. In the first case, the market is entirely covered by just one firm which behaves as a monopolist. However, as we will show later, there can not be an equilibrium in which only one firm serves the whole market. In the second case, i.e., when the second section of the best–response correspondence is downward slopping, the market is entirely covered but the marginal consumer receives zero rents. If firm j unilaterally increases its price, it is profitable for firm i to decrease its own price just enough to cover the market. Hence, the market is entirely covered but the marginal consumer receives zero rents. When the best–response correspondences cross in this range, there is potentially a continuum of equilibria. Finally, the third section of the best-response correspondence is a constant. The other firm's price is so high that charging the monopoly price becomes the best response. When the best–response correspondence cross in that range, each firm charges the monopoly price and part of the market is uncovered. This case arises when the transportation cost coefficients are higher enough relative to $\bar{s}$. For a detail exposition about the derivation of the best–response correspondences see Appendix B.

Technically, there are at most four possible types of equilibria: (i) all the market is covered, it is shared by both firms and the marginal consumer makes positive rents; (ii) all the market is covered, it is shared by both firms and the marginal consumer makes zero rent, (iii) not all the market is covered, and (iv) all the market is covered by just one firm. We shall explore all these possibilities and characterize the conditions under which such equilibria exist.

Case (i). Here the demand function that firm i faces is given by the typical Hotelling demand, i.e., $\left[\frac{p_j^*-p_i^*+t_j}{t_i+t_j}\right]$. The best–response for firm i in this case satisfies the following first–order condition $-\frac{(p_i^*-c)}{t_i+t_j}+\frac{p_j^*-p_i^*+t_j}{t_i+t_j}=0$. Similarly, for firm j we must have that $-\frac{(p_j^*-c)}{t_i+t_j}+\frac{p_i^*-p_j^*+t_i}{t_i+t_j}=0$. Accordingly, the equilibrium prices are uniquely determined by the system of two equations and two unknowns. We have:

$$p_i^* = \frac{2t_j + t_i + 3c}{3} \tag{1}$$

$$p_j^* = \frac{2t_i + t_j + 3c}{3} \tag{2}$$

Recall that we have postulated that the marginal consumer earns positive rents. Therefore, we must also have:

$$0 < \bar{s} - p_i^* - t_i\left[\frac{p_j^* - p_i^* + t_j}{t_i + t_j}\right]$$

$$\text{or } \bar{s} > \frac{(2t_i + t_j)(t_i + 2t_j)}{3(t_i + t_j)} + c$$

Case (ii). By construction, the marginal consumer makes zero rent, so we have $\frac{p_j^*-p_i^*+t_j}{t_i+t_j}=\frac{\bar{s}-p_i^*}{t_i}$. This in turn implies that if firm i increases its price, its demand will be determined by the monopoly demand $\frac{\bar{s}-p_i}{t_i}$, which will not be profitable so long as $-\frac{(p_i-c)}{t_i}+\frac{\bar{s}-p_i}{t_i}\leq 0$ or $\frac{\bar{s}+c}{2}\leqslant p_i$. Now, if firm i decreases its price, its demand will be determined by the Hotelling demand, $\frac{p_j^*-p_i^*+t_j}{t_i+t_j}$. However, it will not be optimal to do so, so long as $-\frac{(p_i^*-c)}{t_i+t_j}+\frac{p_j^*-p_i^*+t_j}{t_i+t_j}\geq 0$. Under the zero rent condition, the latter inequality is satisfied whenever $p_i\leq\frac{\bar{s}(t_i+t_j)+ct_i}{2t_i+t_j}$. Likewise, similar condition holds for firm j. Hence, an equilibrium with full market coverage but zero rents for the marginal buyer will exist if and only if there exists a pair of prices $\left(p_i^*, p_j^*\right)$ that satisfies the following conditions:

$$\bar{s} = \frac{p_i^* t_j + t_i p_j^* + t_i t_j}{t_i + t_j} \tag{3}$$

$$\frac{\bar{s} + c}{2} \leqslant p_i^* \leqslant \frac{\bar{s}(t_i + t_j) + ct_i}{2t_i + t_j} \tag{4}$$

$$\frac{\bar{s} + c}{2} \leqslant p_j^* \leqslant \frac{\bar{s}(t_i + t_j) + ct_j}{2t_j + t_i} \tag{5}$$

Notice that such pair will exist if and only if $\frac{2t_i t_j}{(t_i+t_j)} + c \leq \bar{s} \leq \frac{(2t_i+t_j)(t_i+2t_j)}{3(t_i+t_j)} + c$.

Case (iii). Now suppose that there exists an equilibrium where the market is not fully covered. This implies that the demand function that firm i faces is given by the monopolist one, $\frac{\bar{s}-p_i}{t_i}$. The marginal profit of increasing i's price is given by: $-\frac{(p_i-c)}{t_i} + \frac{\bar{s}-p_i}{t_i}$, which is zero at $p_i = \frac{\bar{s}+c}{2}$. A similar argument applies for j. Hence, we must have,

$$p_i^* = \frac{\bar{s} + c}{2} \tag{6}$$

$$p_j^* = \frac{\bar{s} + c}{2} \tag{7}$$

Nevertheless, by assumption we have that $\frac{p_j^*-p_i^*+t_j}{t_i+t_j} > \frac{\bar{s}-p_i^*}{t_i}$. It follows that we must also have: $\bar{s} < \frac{2t_i t_j}{(t_i+t_j)} + c$.

Case (iv). Suppose that in equilibrium firm i covers the entire market. For this to be an equilibrium, j must have no incentive to lower its price and gain market share, which implies that $-\frac{(p_j^*-c)}{t_i+t_j} + \frac{p_i^*-p_j^*+t_i}{t_i+t_j} \geq 0$. However, we must also have that $p_i^* + t_i \leq p_j^*$, which leads us to $-\frac{(p_j^*-c)}{t_i+t_j} + \frac{p_i^*-p_j^*+t_i}{t_i+t_j} \leq -\frac{(p_j^*-c)}{t_i+t_j} < 0$, implying a contradiction. Hence, no equilibrium exists such that one firm covers the entire market.

Consequently, there are only three types of equilibria for the pricing subgame. The prevailing type of equilibria depends on the value of $\bar{s}$ relative to t_i, t_j and c. Let define,

$$\theta(t_i, t_j, c) \equiv \frac{2t_i t_j}{(t_i + t_j)} + c < \psi(t_i, t_j, c) \equiv \frac{(2t_i + t_j)(t_i + 2t_j)}{3(t_i + t_j)} + c \tag{8}$$

We summarize the above discussion in the following lemma.

Lemma 1. *(a) If* $\bar{s} > \psi(t_i, t_j, c)$*, then the unique equilibrium prices for the final–stage game are given by equation* (1) *and equation* (2) *and the equilibrium profits are given by:*

$$\pi_i(t_i, t_j) = \frac{(2t_j + t_i)^2}{9(t_i + t_j)}$$

(b) If $\theta(t_i, t_j, c) \leq \bar{s} \leq \psi(t_i, t_j, c)$*, the pair of prices* (p_i^*, p_j^*) *forms an equilibrium of the final–stage game if it lies in the interval defined by equations* (4) *and* (5)*.*
(c) Finally, if $\bar{s} < \theta(t_i, t_j, c)$*, the unique price equilibrium for the final–stage game is given by equations* (6) *and* (7) *and the equilibrium profits are given by:*

$$\pi_i(t_i, t_j) = \frac{(\bar{s} - c)^2}{4t_i}$$

4. The equilibrium in the two–stage game

Recall that the idea behind subgame perfection is that firms will move at each stage assuming that a play in this stage will correspond to an equilibrium for the payoffs prevailing at h^{k+1}. Accordingly, in stage one players know that the outcome that will prevail in stage–two, i.e., in the pricing subgame, will correspond to one of those outcome described in Lemma (1). Nonetheless, before characterizing the equilibrium in the two–stage game, we present the following lemma.

Lemma 2. *Suppose the pair* (t_i^*, t_j^*) *are part of a subgame perfect Nash equilibrium for* Γ*. Then* (t_i^*, t_j^*) *must satisfy,*

$$\theta(t_i^*, t_j^*, c) \leqslant \bar{s} \leqslant \psi(t_i^*, t_j^*, c)$$

where $\theta(t_i^*, t_j^*, c)$ *and* $\psi(t_i^*, t_j^*, c)$ *are defined in* (8)*.*

Proof. First suppose that $\bar{s} > \psi(t_i^*, t_j^*, c)$. By Lemma (1) part (a), the continuation equilibrium will define the following profit function,

$$\pi_i(t_i^*, t_j^*) = \frac{(2t_j^* + t_i^*)^2}{9(t_i^* + t_j^*)}$$

Clearly, there always exists an $\epsilon > 0$, such that firm i can make strictly higher profits picking a transportation cost coefficient equal to $t_i^* + \epsilon$ and such that $\bar{s} > \psi(t_i^* + \epsilon, t_j^*, c)$. Second, suppose that $\bar{s} < \theta(t_i^*, t_j^*, c)$. By Lemma (1), part (c), the continuation equilibrium will determine the following payoff function:

$$\pi_i(t_i^*, t_j^*) = \frac{(\bar{s} - c)^2}{4t_i^*}$$

In this case firm i will always do better selecting a t_i strictly less than t_i^*. Indeed, for any $\epsilon > 0$ such that $\bar{s} < \theta(t_i^* - \epsilon, t_j^*, c)$, $\pi_i(t_i^* - \epsilon, t_j^*) > \pi_i(t_i^*, t_j^*)$. Thus, whenever $\bar{s} > \psi(t_i^*, t_j^*, c)$ or $\bar{s} < \theta(t_i^*, t_j^*, c)$, there is an incentive for either firm to deviate from (t_i^*, t_j^*), which contradicts the fact that (t_i^*, t_j^*) is part of a subgame perfect Nash equilibrium. □

Basically what Lemma (2) says is that an equilibrium of the two–stage game must be of the form describe in part (b) of Lemma (1). Therefore, in equilibrium, firms do not engage in price competition *à la Hotelling*, the portion of the market served by each firm never overlaps its competitor's and the marginal consumer makes zero rent. Formally, we show existence of such equilibria in the following Theorem.

Theorem 1. *For the two–stage game* Γ *described in section 2, there are multiple subgame perfect Nash equilibria, denoted* $\left\{(t_i^*, p_i^*(\cdot,\cdot)), \left(t_j^*, p_j^*(\cdot,\cdot)\right)\right\}$, *which satisfy the following conditions:*

$$\bar{s} = \psi(t_i^*, t_j^*, c) = \frac{2t_i^{*2} + 2t_j^{*2} + 5t_i^* t_j^*}{3(t_i^* + t_j^*)} + c \tag{9}$$

$$p_i^*(t_i^*, t_j^*) = \frac{2t_j^* + t_i^* + 3c}{3} \tag{10}$$

$$p_j^*(t_i^*, t_j^*) = \frac{2t_i^* + t_j^* + 3c}{3} \tag{11}$$

Proof. We need to show that there exists a set of pricing strategies $(p_i^*(.,.); p_j^*(.,.))$ such that for every t_i and t_j, $(p_i^*(t_i, t_j); p_j^*(t_i, t_j))$ forms a Nash equilibrium for the corresponding pricing subgame and that for all i,

$$\pi_i\left([\widehat{t}_i, p_i^*(\widehat{t}_i, t_j^*)]; [t_j^*, p_j^*(\widehat{t}_i, t_j^*)]\right) \leqslant \pi_i\left([t_i^*, p_i^*(t_i^*, t_j^*)]; [t_j^*, p_j^*(t_i^*, t_j^*)]\right) = \frac{(2t_j^* + t_i^*)^2}{9(t_i^* + t_j^*)} \; \forall\, \widehat{t}_i$$

Suppose that i deviates and decreases the transportation cost coefficient to $\widehat{t}_i < t_i^*$, such that $\psi(\widehat{t}_i, t_j^*, c) < \bar{s} = \psi(t_i^*, t_j^*, c)$. The only admissible pair of prices that forms a Nash equilibrium for the continuation game is given in part (a) of Lemma (1). These prices yield profits of

$$\pi_i(\widehat{t}_i, t_j^*) = \frac{(2t_j^* + \widehat{t}_i)^2}{9(\widehat{t}_i + t_j^*)} < \pi_i(t_i^*, t_j^*) = \frac{(2t_j^* + t_i^*)^2}{9(t_i^* + t_j^*)}$$

Now suppose that i deviates and increases t_i to $\widehat{t}_i > t_i^*$ such that $\bar{s} < \theta(\widehat{t}_i, t_j^*, c)$. In this case, the pair of prices that forms a Nash equilibrium for the continuation subgame is determined in part (c) of Lemma (1). This prices yield profits equal to $\frac{(\bar{s}-c)^2}{4\widehat{t}_i}$. Notice that:

$$\bar{s} < \theta(\widehat{t_i}, t_j^*, c) \Rightarrow \frac{(\bar{s}-c)}{2\widehat{t_i}} < \frac{t_j^*}{\left(\widehat{t_i} + t_j^*\right)}$$

$$\text{while, } \bar{s} = \psi(t_i^*, t_j^*, c) \Rightarrow \frac{(\bar{s}-c)}{2} = \frac{\left(2t_i^* + t_j^*\right)\left(t_i^* + 2t_j^*\right)}{3(2t_i^* + 2t_j^*)}$$

$$\text{so, } \frac{(\bar{s}-c)^2}{4\widehat{t_i}} < \frac{t_j^*}{\left(\widehat{t_i} + t_j^*\right)} \frac{\left(2t_i^* + t_j^*\right)\left(t_i^* + 2t_j^*\right)}{3(2t_i^* + 2t_j^*)} \leq \frac{(2t_j^* + t_i^*)^2}{9(t_i^* + t_j^*)}$$

The last inequality requires some algebra, but it establishes that when $\bar{s} < \theta(\widehat{t_i}, t_j^*, c)$, firm i cannot profitably increase t_i up to $\widehat{t_i}$.

Now suppose that i deviates and increases t_i so that $\theta(\widehat{t_i}, t_j^*, c) < \bar{s} < \psi(\widehat{t_i}, t_j^*, c)$. Then, there exists possibly a continuum of equilibrium prices for the continuation of the game. We propose one of these possible continuation equilibrium which is such that player i will be worse off. We assume that if firm i deviates, firm j keeps its price unchanged, which is equivalent to claim that:

$$p_j^*(\widehat{t_i}, t_j^*) = \frac{2t_i^* + t_j^* + 3c}{3} = \frac{\bar{s}(t_i^* + t_j^*) + ct_j^*}{2t_j^* + t_i^*} \leq \frac{\bar{s}(\widehat{t_i} + t_j^*) + ct_j^*}{2t_j^* + \widehat{t_i}}$$

The last inequality guarantees that j's price remains constant within the admissible interval stated in equations (4) and (5). Since j's price is unchanged so will be its market share. As a best response, i captures the same market by lowering its price in order to offset the raise in the transportation cost. Overall, its profits can only be lowered. Hence, firm i cannot profitable deviate from t_i^*. A symmetric argument applies for player j. □

Theorem 1 ensures the existence of a symmetrical equilibrium for the two–stage game Γ. When $t_i^* = t_j^* = t^*$, conditions (9), (10) and (11) in Theorem 1 become,

$$\bar{s} = \frac{3}{2}t^* + c \tag{12}$$

$$p_i^* = p_j^* = \frac{2\bar{s} + c}{3} \tag{13}$$

First, notice that in the 'traditional' Hotelling model with firms located at the two ends of the linear city, a necessary condition to ensure the existence of a competitive equilibrium is that t be equal to or less than $2(\bar{s} - c)/3$. Under this condition it is also ensured that the market will be totally covered. However, if transportation cost coefficients are made endogenous, t^* will always equal $2(\bar{s}-c)/3$, regardless the particular value of $\bar{s}$. Likewise, with endogenous transportation cost coefficients the marginal consumer (the one who is indifferent between buying the good at either firm) will obtain zero rent in equilibrium. On the contrary, in the 'traditional' Hotelling model, this consumer may perceive a positive rent on condition that her reservation price is strictly higher than $\frac{3}{2}t + c$. This latter result is ruled out when transportation cost coefficients are endogenous.

5. Concluding Remarks

In this paper we study a model of spatial competition in which firms are entitle to choose the degree of general purposeness (or focus) of their products by selecting a transportation cost coefficient before competing in prices. The game takes place in two stages. In the first stage, firms simultaneously and independently select transportation costs, whereas in the second stage firms compete non cooperatively in prices. The main insight of the model is that firms choose to follow, in equilibrium, a 'specialist' strategy. In terms of the degree of purposeness, this means that firms select a high transportation cost coefficient (which is equivalent to compete with 'targeted goods') in order to increase the disutility that consumers need to bear when they do not find their preferred good. It is in this sense that a higher transportation cost coefficient is similar to more specific products designed for each consumer in particular. This in turn allows firms to perfectly segment the market and to exert their monopoly power in their corresponding market segment in order to capture the rent otherwise enjoyed by the marginal consumer through product design and not through price competition. As a consequence of this, a second insight of the model is the particular market configuration prevailing in equilibrium. This structure is such that firms act as local monopolist but cover the whole market. In order to accomplish this result, it is not necessary to rely on a particular value of the consumer's reservation price because firms are able to adjust the degree of purposeness of their products to the particular characteristics of the market and to ensure that this will be always covered in equilibrium.

When the model developed in this paper is compared with the 'traditional' Hotelling model with maximal product differentiation, an interesting difference arises. For identical transportation costs, our model predicts that in equilibrium the transportation cost will equal $2(\bar{s}-c)/3$. On the contrary, in the traditional Hotelling model it is required that t be at most equal to $2(\bar{s}-c)/3$ to ensure total market coverage. Furthermore, in the latter model the marginal consumer will perceive a strictly positive rent on condition that her reservation price is strictly higher than $\frac{3}{2}t + c$. Thus, the equilibrium reached in the spatial model with endogenous transportation cost coefficients is inefficient when compared to the equilibrium of the traditional Hotelling model so as in the model with endogenous transportation coefficients the marginal consumer will never earn a positive rent.

All the insights discussed above have been established in a particular framework that calls for discussion. Thus, in our model firms will never reduce their transportation cost coefficients. Instead, they will always want to increase these coefficients in order to extract as much of the marginal consumer's surplus as possible. Hence, the model developed in this paper suggests that no firm will invest in any technology that help to reduce transportation (or disutility) costs. Yet, it is reasonable to conjecture that firms might want to reduce their transportation cost coefficients on condition that other mechanism to soften price competition is available. We could think, for instance, of capacity constraints as such mechanism. Indeed, Boccard and Wauthy (2000) have shown that the introduction of capacity constraints in the Hotelling model may completely rule out price competition. Thus, a possible extension to the present model appears to be the combination of both endogenous transportation costs and capacity constraints. We leave this important issue to be developed by further research.

Appendix A. The demand function

As shown in figure 1, the demand function can be represented using three different areas within the price space. For those price pairs inside area **H** ($t_i\text{CB}t_j$), the relevant demand function is given by $\left[\frac{p_j - p_i + t_j}{t_i + t_j}\right]$, $i = 1, 2$. This is so because inside this area prices are such that the consumer located at x^* obtains a positive surplus when buying the good and the consumer located at the opposite side of firm i (j) obtains a surplus that is strictly less than the one he would obtain from buying the good at the nearest firm. When either of these conditions is violated, firms become monopolist. However, the monopoly condition (and hence the relevant demand) differs depending on whether prices fall within area LM or area M. Inside area LM, the relevant demand is given by $\frac{\bar{s} - p_i}{t_i}$, $i = 1, 2$, because the consumer located at x^* is better off buying zero units of the good and so only the nearest customers to each firm are willing to make a purchase. On the contrary, when prices fall inside area M all consumers are willing to buy the good from just one firm because the consumer located at the opposite side of firm i (j) obtains a surplus strictly greater than the one he would obtain from buying the good at firm j (i). Thus, the relevant demand in this case is equal to one and the market is served by just one firm.

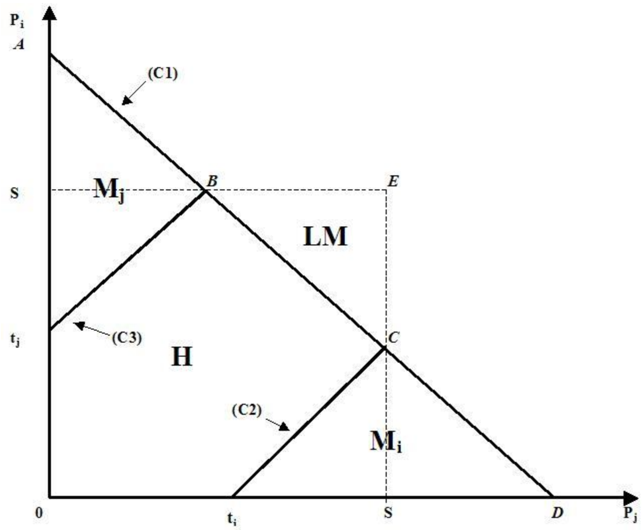

FIGURE 1. The demand function.

APPENDIX B. THE BEST–RESPONSE CORRESPONDENCES

The best responses of player i can be written as follows:

$$B_i(p_j, t_i, t_j) = \arg\max_{p_i} \left[(p_i - c) \max \left[\min \left\{ \frac{p_j - p_i + t_j}{t_i + t_j}; \frac{\bar{s} - p_i}{t_i}; 1 \right\}; 0 \right] \right]$$

Given the kinked structure of the demand function, the best–response correspondence of player i is formed by three segments. For those prices within area **H** in Figure 1, firms optimally response to each other using the familiar Hotelling best response correspondence. Now, when prices are on the boundaries or outside area **H**, three different cases arise depending on the relationship among $\bar{s}$, c, t_i and t_j. First, suppose that $\bar{s} \geqslant 2t_i + t_j + c$. Given this inequality, for any p_j in the interval $]2t_i + t_j + c, \bar{s}]$, the consumer located at the opposite side of firm i's obtains a greater surplus buying the good at firm i than the surplus she would obtain buying the good at firm j. Consequently, firm i can cover the whole market and its the best response is to set a price equal to $p_i(p_j) = p_j - t_i$. When the competitor's price, p_j, becomes equal to or greater than $\bar{s}$, no consumer in the city will be willing to make a purchase at firm j and therefore, firm i becomes the unique firm serving the market. Its best response in this case is simply the monopolistic price $p_i^m = \bar{s} - t_i$. Hence, the best response correspondence for firm i when $\bar{s} \geqslant 2t_i + t_j + c$ is given by,

$$B_i(s_{-i}) = \begin{cases} \frac{p_j+t_j+c}{2} & \text{if } p_j \in [0, 2t_i+t_j+c]; \\ p_j - t_i & \text{if } p_j \in]2t_i+t_j+c, \bar{s}]; \\ \bar{s} - t_i & \text{if } p_j \in]\bar{s}, +\infty[. \end{cases} \qquad (B_{i,1})$$

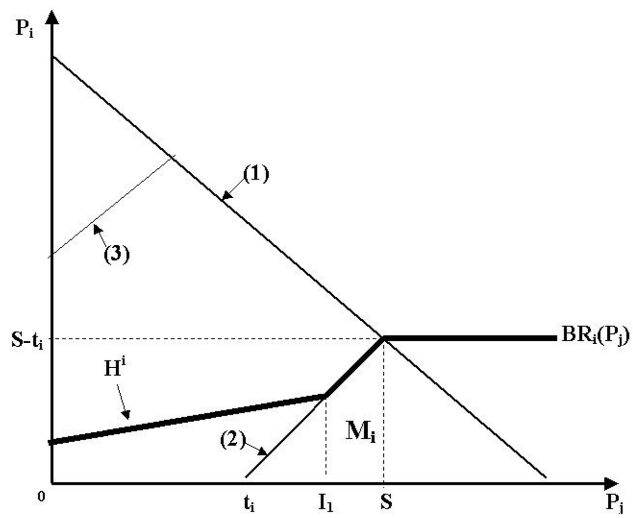

FIGURE 2. The best response correspondence.

Suppose now that $2t_i + t_j + c \geqslant \bar{s} > 2t_i + c$. When this last inequality holds, the marginal consumer located at x^* obtains a rent exactly equal to zero. If firm j unilaterally increases its price over $\widehat{p_j} = \frac{2\bar{s}(t_i+t_j)-2t_it_j-t_j^2-ct_j}{2t_i+t_j}$, this marginal consumer is better off buying zero unit of the good, which in turns implies that the market is partially uncovered. However, the best response of firm i if $p_j > \widehat{p_j}$ is to *decrease* its price in order to serve the uncovered portion of the market and thus, obtains higher profits. Similar to the previous case, when p_j is equal to or greater than $\bar{s}$, no consumer will do business with firm j and firm i becomes a pure monopoly. Hence, the best response function for firm i (the bold line in figure 3) when $2t_i + t_j + c \geqslant \bar{s} > 2t_i + c$ is given by,

$$B_i(s_{-i}) = \begin{cases} \frac{p_j+t_j+c}{2} & \text{if } p_j \in \left[0, \frac{2\bar{s}(t_i+t_j)-2t_it_j-t_j^2-ct_j}{2t_i+t_j}\right]; \\ \bar{s}(\frac{t_i+t_j}{t_j}) - \frac{t_i}{t_j}(p_j+t_j) & \text{if } p_j \in \left]\frac{2\bar{s}(t_i+t_j)-2t_it_j-t_j^2-ct_j}{2t_i+t_j}, \bar{s}\right]; \\ \bar{s} - t_i & \text{if } p_j \in]\bar{s}, +\infty[. \end{cases} \qquad (B_{i,2})$$

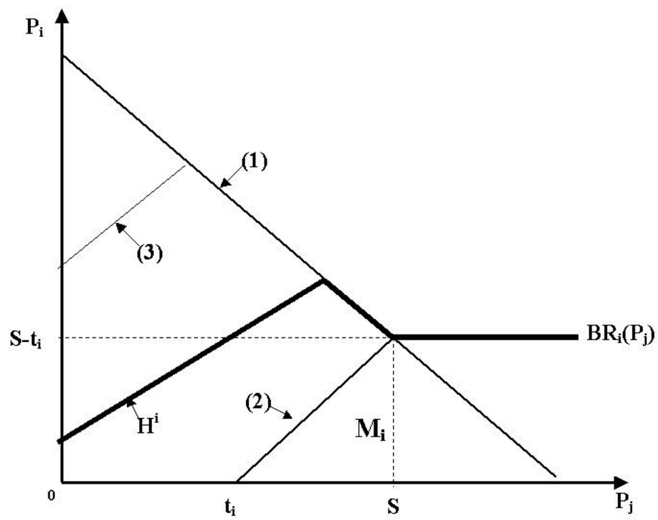

FIGURE 3. The best response correspondence.

Finally, suppose that $2t_i + t_j + c > 2t_i + c > \bar{s}$. In this case, the fact that $\bar{s}$ is strictly less than $2t_i + c$ implies that no firm will cover the whole market. Since the procedure to derive the best–response correspondence is similar to the previous cases, we save all the intermediate steps. The best response function when $2t_i + t_j + c > 2t_i + c > \bar{s}$ is given by,

$$B_i(s_{-i}) = \begin{cases} \frac{p_j+t_j+c}{2} & \text{if } p_j \in \left[0, \frac{2\bar{s}(t_i+t_j)-2t_it_j-t_j^2-ct_j}{2t_i+t_j}\right]; \\ \bar{s}(\frac{t_i+t_j}{t_j}) - \frac{t_i}{t_j}(p_j+t_j) & \text{if } p_j \in \left]\frac{2\bar{s}(t_i+t_j)-2t_it_j-t_j^2-ct_j}{2t_i+t_j}, \frac{\bar{s}(2t_i+t_j)-t_jc-2t_it_j}{2t_i}\right]; \\ \frac{\bar{s}+c}{2} & \text{if } p_j \in \left]\frac{\bar{s}(2t_i+t_j)-t_jc-2t_it_j}{2t_i}, +\infty\right[; \end{cases} \qquad (B_{i,3})$$

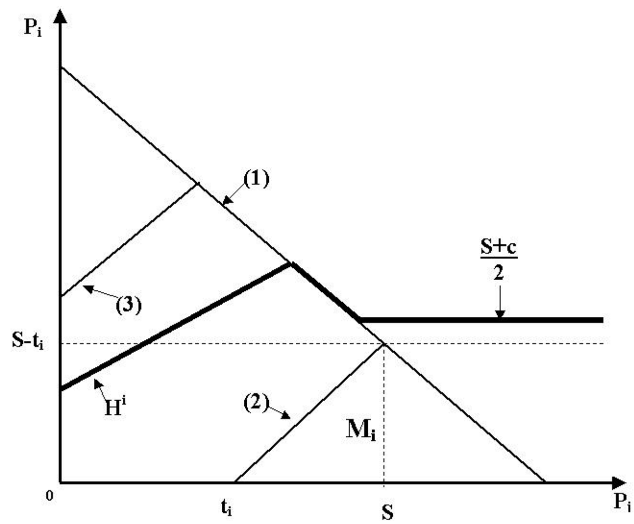

Figure 4. The best response correspondence.

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