INVESTIGACIÓN / RESEARCH

GARCH Processes and Value at Risk: An Empirical Analysis for Mexican Interest Rate Futures[1]

Procesos GARCH y Valor en Riesgo: Un Análisis Empírico de Futuros de Tasas de Interés Mexicanas

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ABSTRACT. In this research paper GARCH processes are applied in order to estimate Value at Risk (VaR) for an interest rate futures portfolio. According to several documents in the literature, GARCH models tend to overestimate VaR because of volatility persistence. The main objective here is to put to test if GARCH models actually overestimate VaR. The analysis is carried out for several time-horizons for the above mentioned asset, which has trading at the Mexican Derivatives Exchange. To analyze the VaR with time horizons of more than one trading day Real-World Densities (RWD) are estimated applying GARCH processes. The results show that GARCH models are relatively accurate for time horizons of one trading day. However, the volatility persistence captured by these models is reflected with relatively high VaR estimates for longer time horizons. In terms of Risk Management this is considered undesirable given that not-optimal amounts of capital must be set aside in order to meet Minimum Capital Risk Requirements for futures portfolios. These results have also implications for short-term interest rate forecasts given that RWD are estimated.

Keywords: Bootstrapping, GARCH, interest rates, Mexico, Value at Risk, volatility persistence.

RESUMEN. En el presente trabajo de investigación se utilizan procesos GARCH para estimar el valor-en-riesgo (VaR) de un portafolio hipotético de futuros de tasas de interés. De acuerdo con algunos documentos en la literatura (Brooks, et. al. 2000), los modelos GARCH tienden a sobreestimar el VaR debido a que capturan la persistencia en la volatilidad. El principal objetivo del presente trabajo es poner a prueba si los modelos GARCH en realidad sobreestiman el VaR. El análisis se lleva a cabo para diferentes horizontes en el tiempo para el activo previamente mencionado, el cual tiene negociación en el Mercado Mexicano de Derivados (Mexder). Para analizar el VaR con horizontes de tiempo de más de un día de negociación (trading day) densidades del mundo-real son estimadas con procesos GARCH y simulaciones Bootstrapping. Los resultados muestran que los modelos GARCH son relativamente certeros para horizontes de un día de negociación. Sin embargo, la volatilidad persistente capturada por este tipo de modelos se refleja con estimados de VaR relativamente altos para horizontes de tiempo mayores (Ej. de diez días de negociación ó más). En términos de análisis de riesgos lo anterior es considerado subóptimo ya que cantidades innecesarias de capital tendrían que destinarse para cubrir los requerimientos mínimos de capital en riesgo (Minimum Capital Risk Requirements). Lo anterior para una posición (corta ó larga) en un portafolio de futuros de tasas de interés. Los métodos aquí explicados pueden servir para pronosticar tasas de interés, ya que estas se estiman a través de estimaciones de densidades del mundo-real. La investigación aquí realizada tiene implicaciones para bancos centrales, ya que se obtienen predicciones de distribuciones de tasas de interés y se analizan estimaciones de VaR. Esto último es relevante para un Banco Central considerando su función de supervisor financiero.

Palabras clave: GARCH, Mexico, persistencia en la volatilidad, remuestreo, tasas de interés, Valor en Riesgo.

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1. INTRODUCTION

Nowadays it is important to measure financial risks in order to make better-informed decisions relevant to risk management. It is well documented that volatility is a measure of financial risk. Measuring financial volatility of asset prices is a way of quantifying potential losses due to financial risks. An important tool for this measure is to estimate volatility-based Value at Risk (VaR). Nowadays, there are several methods applied in order to obtain a volatility-based Value at Risk. Among the most popular ones are the use of ARCH models.

The main objective of this paper is to analyze if Autoregressive Conditional Heteroscedasticity (ARCH-type) models are accurate to predict risks caused by interest rate volatility from a Value at Risk perspective. The idea is to analyze if volatility persistence inherent in this type of models affects VaR. Volatility persistence in this project refers to the financial volatility that takes a long time to die away. This is done by considering a theoretical portfolio of interest rate (Cetes 91-day and TIIE 28-day) futures. VaR is estimated using ARCH-type models and then their accuracy is formally tested with back-testing (Kupiec: 1995, Jorion: 2000, 2001). The procedure is to find out how accurate is the VaR with daily interest rate futures observations. The time horizons considered are from on trading day up to six months or equivalent in trading days. For one trading day a parametric approach is applied. For ten trading days and more Bootstrapping simulations (Enfron: 1982) are carried out (non-parametric approach). If the number of daily violations or 'exceptions' is reasonable according to VaR models performance criteria (Jorion: 1998), then the models are considered accurate. Otherwise, the ARCH-type models are rejected. According to Pérignon, Deng and Wang (2006) banks normally over estimate VaR. The n-day forecast horizon is also interpreted as the probability that future interest rate will be within certain statistical confidence interval i.e. the 95% confidence interval VaR. It is expected that these results could have implications for forecasts about the future range of Mexican interest rates.

The layout of this paper is as follows. The literature review is presented in Section 2. The motivation and contribution of this work are presented in Section 3. The models are explained in Section 4. Data is detailed in Section 5. Section 6 presents the descriptive statistics. The results are analysed in Section 7. Finally, Section 8 concludes.

2. LITERATURE REVIEW

Historical volatility is described by Brooks (2002) as simply involving calculation of the variance or standard deviation of returns in the usual statistical way over some long period (time frame). This variance or standard deviation may become a volatility forecast for all future periods (Markowitz: 1952). However, in this type of calculation there is a drawback. This is because volatility is assumed constant for a specified period of time. Nowadays, it is well known that financial prices have time-varying volatility i.e. volatility changes through time (the volatility that it is considered here is the conditional volatility of a financial asset and not necessarily the unconditional one. I am thankful to Victor Guerrero for asking me to clarify this point). It is well documented that non-linear ARCH models can provide accurate estimates of time-varying price volatility. Just to mention a few papers see for example, Engle (1982), Taylor (1985), Bollerslev, Chou and Kroner (1992), Ng and Pirrong (1994), Susmel and Thompson (1997), Wei and Leuthold (1998), Engle (2000), Manfredo et al. (2001), etc (For an excellent survey about applications of ARCH models in Finance the reader can refer to Bollerslev, Chou and Kroner (1992)).

Nonetheless, there is a growing literature of the implications of non-linear dynamics for financial risk management (Brock et al.:1992; Hsieh: 1993). In the light of this topic some researchers have extended the work for the application of time-varying volatility models, specifically ARCH-type models, in VaR estimations (Brooks, Clare and Persand: 2000; Manfredo: 2001; Engle: 2003; Giot: 2005; Mohamed: 2005; among others). Most of these findings enhance the use of time-varying models in risk management applications using VaR. Even though, there are several research papers, which used these types of models for financial time series there is, however, no works that have analysed VaR for interest rate futures in an emerging economy. This is considered a 'gap' in the literature.

3. MOTIVATION AND CONTRIBUTION

Previous works have applied non-linear models within a VaR framework in order to estimate Minimum Capital Risk Requirements (MCRRs) (Hsieh: 1991; Brooks, Clare and Persand: 2000). MCRR is defined as the minimum amount of capital needed to successfully handle all but a pre-specified percentage of possible losses (Brooks, Clare and Persand: 2000). This concept is relevant to banks and bank regulators. For the latter it is important to require banks to maintain enough capital so banks could absorb unforeseen losses. These regulatory practices go back to the original Basle Accord of 1988. Even tough there is a broad agreement about the need of MCRRs there is, however, significantly less agreement about the method to calculate them. According to Brooks, Clare and Persand (2000) the most well known methods are the Standard/International Model Approach of the Basle Accord (1988), the Building-Block Approach of the EC Capital Adequacy Directive (CAD), the Comprehensive Approach of the Securities Exchange Commission (SEC) of the US, the Pre-commitment Approach of the Federal Reserve Board (FED) and the Portfolio Approach of the Securities and Futures Authority of the UK. By estimating the VaR of their financial portfolios banks are able to calculate the amount of MCRRs needed to meet bank supervision requirements. According to Basel Bank Supervision Requirements, banks have to hold capital (as a precautionary action) at least three times the equivalent to the VaR for a time horizon of 10 trading days at the 99% confidence level.

In this project the works of Hsieh (1991) and Brooks, Clare and Persand (2000) are extended. The extension here is that MCRRs are estimated for futures contracts that have not been applied for this type of analysis and that the null hypothesis that ARCH-type models overestimate VaR is tested. This also has implications for interest rate forecasts. By estimating Real-World densities it is possible to have an idea of future interest rate range-levels with certain statistical confidence. For example, if a 95% confidence level VaR with a time horizon of one month is applied, it is possible to quantify the range of possible interest rates in one month with 95% statistical certainty. Also, it is possible to quantify what are the chances of observing those extreme values i.e. one in twenty (those outside the 95% interval in a parametric and non-parametric distribution).

These findings contribute with new knowledge to the existing academic literature on the use of time-varying volatility models in VaR estimates. The results could be for the interest of agents involved in making risk management decisions related to interest rate forecasts. These groups of persons could be private bankers, policy makers, investors, futures traders, central bankers, academic researchers, among others.

4. THE MODELS

4.1 GARCH Specification

The volatility of the time series under analysis is estimated with historical data. It is known that ARCH models (Engle: 1982) are accurate estimators of time-varying volatility. A well known model within the family of ARCH models is the univariate Generalized Autoregressive Conditional heteroscedasticity, GARCH(p, q) model. This model is estimated applying the standard procedure as explained in Bollerslev (1986) and Taylor (1986) (the ARCH-type models presented in this paper were estimated using Eviews computer language). The formulae for the GARCH(p, q) are presented below. For the model there are two main equations. These are the mean equation and the variance equation:

mean equation,

$$\Delta y_t = \mu + e_t \qquad e_t \mid I_{t-1} \sim N(0, \sigma^2_t), \tag{1}$$

and the variance equation,

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{2}$$

Where: Δy_t = first differences of the natural log (logs) of the series under analysis at time *t* (the interest rate spot or futures-index), e_t is the error term at time *t*, I_{t-1} is the information set at time *t*-1, s^2_t = variance at time *t* and *t-j* for s$^2_{t-j}$. m, w, $_i$ $_j$ are parameters and *N*(0, s^2_t) is for the assumption that the log returns are normally distributed. In other words, assuming a constant mean m (the mean of the series y_t) the distribution of e_t is assumed to be Gaussian with zero mean and variance s^2_t. The parameters are estimated using maximum likelihood methodology applying the Marquardt algorithm This algorithm modifies the Gauss-Newton algorithm by adding a correction matrix to the Hessian approximation. This allows to handle numerical problems when the outer products are near singular thus, increases the chance of improving convergence of the parameters. The objective log-likelihood function to be maximized is the following:

$$\ln L(\theta) = -\frac{1}{2}\sum_{t=1}^{n}\left[\ln(2\pi) + \ln\left(\sigma_t^2(\theta)\right) + z_t^2(\theta)\right], \tag{3}$$

where θ is the set of parameters ($\mu, \omega, \alpha_i, \beta_i$) estimated that maximize the objective function ln $L(\theta)$. z_t represents the standardized residual calculated as $\frac{\Delta y_t - \mu}{\sqrt{\sigma_t^2}}$.

The rest of the notation is the same as expressed previously.

Considering that the assumption of normality in the residuals stated above does not hold (as it is common with financial high frequency price data), the Bollerslev and Wooldridge (1992) methodology is used in order to estimate consistent standard errors. With this method the results have robust standard errors and covariance. The estimated coefficients are reliable once they are positive, statistically significant and the sum of the $\alpha + \beta < 1$ (otherwise the series are considered explosive or equivalently non-mean reverting, Taylor: 1986).

4.2 The VaR model

The VaR is a useful measure of risk (Value at Risk is normally abbreviated as VaR. The small letter *a* differentiates this abbreviation to that of Vector Autoregressive Models, which are usually abbreviated as VAR (with a capitol A)). It was developed in the early 1990s by the corporation JPMorgan. According to Jorion (2001) 'VaR summarizes the expected maximum loss over a target horizon with a given level of confidence interval.' Even though it is a statistical figure, most of the times is presented in monetary terms. The intuition is to have an estimate of the potential change in the value of a financial asset resulting from systemic market changes over a specified time horizon (Mohamed: 2005). It is also normally used to obtain the probability of losses for a financial portfolio of futures contracts. Assuming normality, the VaR estimate is relatively easy to obtain from GARCH models. For example, for a one trading day 95% confidence interval VaR the estimated GARCH standard deviation (for the next day) is multiplied by 1.645. If the standard deviation forecast is, lets say, 0.0065, the VaR is approximately 1.07%. To interpret this result it could be said that an investor can be 95% sure that he or she will not lose more than 1.07% of asset or portfolio value in that specific day. However, a problem with a parametric approach is that if the observed asset returns depart significantly from a normal distribution the applied statistical model may be incorrect to use (Dowd: 1998).

So, as it was said, when using VaR models it is necessary to make an assumption about the distribution of the returns. Although normality is often assumed, it is known in practice that for price returns series normality is highly questionable (Mandelbrot: 1963, Fama: 1965, Engle: 1982, 2003).

For time horizons of more than one trading day (ten, thirty, ninety and one hundred and eighty trading days), the bootstrapping methodology of Enfron (1982) will be applied The bootstrap is a resampling method for inferring the distribution of a statistic, which is derived by the data in the population sample. This is normally estimated by simulations. It is said to be a nonparametric method given that it does not draw repeated samples from well-known statistical distributions. On the other hand, Monte Carlo simulations draw repeated samples from assumed distributions. In this research project the bootstrap methodology was implemented using Eviews computer language. The fact that the returns of the series are non-normally distributed motivates the use of a non-parametric procedure as the bootstrapping. The procedure used in Hsieh (1993) and Brooks, Clare and Persand (2000) is considered here. In the latter they empirically tested the performance of that VaR model for futures contracts traded in the London International Financial Futures Exchange (LIFFE) (these futures contracts were the FTSE-100 stock index futures contract, the Short Sterling contract and the Gilt contract). A similar paradigm is applied here for interest rate-indexed (interest rate) futures contracts. Thus, a hypothetical portfolio of interest rate futures is considered and MCRRs will be estimated. These estimated MCRRs values for the interest rate portfolio are compared to the observed (historical) interest rates. This analysis allows to evaluate how accurate are the ARCH-type models in terms of estimating MCRRs for interest rate-indexed futures. Yet, another objective is to analyze the performance of these in terms of how accurate are they for providing an upper threshold for interest rates i.e. what are the statistical chances that interest rate will be high enough to be outside the upper (positive) confidence interval. In order to calculate an appropriate VaR estimate it is necessary to find out the maximum loss that a position might have during the life of the futures contract. In other words, by replicating with the bootstrap the daily values of a long futures position it is possible to obtain the possible loss during the sample period. This will be obtained with the lowest replicated value. The same reasoning applies for a short position. But in that case the highest possible loss will be obtained with the highest replicated value As it is well known in futures market mechanics decreases in futures prices mean losses for long positions and increases in futures prices mean losses for short positions. Following Brooks, Clare and Persand (2000) and Brooks (2002) the formulae is as follows. The maximum loss (*L*) is given by

$$L = (P_0 - P_1) \times \text{Number of contracts} \quad (4)$$

where P_0 represents the price at which the contract is initially bought or sold; and P_1 is the lowest (highest) simulated price for a long (short) position, respectively, over the holding period. Without loss of generality it is possible to assume that the number of contracts held is one. Algebraically, the following can be written,

$$\frac{L}{P_0} = \left(1 - \frac{P_1}{P_0}\right) \quad (5)$$

Given that P_0 is a constant, the distribution of *L* will depend on the distribution of P_1. It is reasonable to assume that prices are lognormally distributed (Hsieh: 1993) i.e. the log of the ratios of the prices are normally distributed The log of the ratios of the prices can be represented as $\ln(P_1/P_0)$. However, this assumption is not considered here. Instead the log of the ratios of the prices is transformed into a standard normal distribution following JPMorgan Risk-Metrics (1996) methodology. This is done by matching the moments of the log of the ratios of the prices' distribution to a distribution from a set of possible ones known (Johnson: 1949). Following Johnson (1949) a standard normal variable can be constructed by subtracting the mean from the log returns and then dividing it by the standard deviation of the series,

$$\frac{\ln\left(\frac{P_1}{P_0}\right) - \mu}{\sigma} \quad (6)$$

The expression above is approximately normally distributed. It is known that the 5% lower (upper) tail critical value is -1.645 (1.645). To find the fifth percentile then the following applies,

$$\frac{\ln\left(\frac{P_1}{P_0}\right) - \mu}{\sigma} = \pm 1.645 \quad (7)$$

Cross-multiplying and taking the exponential the case for the long position is,

$$\frac{P_1}{P_0} = \exp[-1.645\sigma + \mu] \quad (8)$$

From Equation 5 the following can be expressed,

$$\frac{L}{P_0} = 1 - \exp[-1.645\sigma + \mu] \quad (9)$$

when the maximum loss for the long position is obtained. For the case of finding the maximum possible loss for the short position the following formula applies,

$$\frac{L}{P_0} = \exp[1.645\sigma + \mu] - 1 \quad (10)$$

The MCRRs of the short position can be interpreted as an upper threshold for interest rate. This will be the threshold of more interest given that in the Mexican economy it was common to observe increases in interest rates. MCRRs for both positions are reported in this paper.

The simulations were performed in the following way. The GARCH model was estimated with the bootstrap using the standardized residuals from the whole sample (instead of residuals taken from a normal distribution as it was written in Equation 1). The interest rate variable was simulated, with the bootstrap as well, for the relevant time horizon (10, 30, 90 and 180 trading days) with 10,000 replications. The formula used was $r_1 = r_{t-1}\, e^{returnT}$ (where interest rate is defined as *r* and could be the futures or spot price. The rest of the notation is the same as specified above). From the interest rate simulations the maximum and minimum values were taken in order to have the MCRRs for the short and long positions respectively.

5. DATA SOURCES

The data consists of daily spot and futures closing prices of the interest rate obtained from Banco de México and MEXDER respectively. The data was downloaded from both institutions' Web Pages (Banco de México's Web page is http://www.banxico.org.mx (the Web page is also available in English). The MEXDER web page is http://www.mexder.com.mx). Two types of interest rates are considered: Cetes 91-days and TIIE 28-days. The first one is calculated from Mexican Government Bonds and the second one is an equilibrium rate calculated according to Mexican commercial banks borrowing and lending transactions. The sample size is 951 daily observations from 1st January 2003 to 29th September 2006. The sample period was chosen according to availability of interest rate futures data and high volume of trading. 2004 was a year of relatively high trading for Mexican interest rate futures. According to the Futures Industry Association these types of contracts were rank fifth in the world in terms of volume of trading. In other words, these are highly liquid futures contracts. The interest rate contracts used here are the closest ones to maturity. They have delivery dates for up to ten years. The MEXDER is relatively new compared to other derivatives exchanges around the world. It began operations in 1998.

6. DESCRIPTIVE STATISTICS

This section presents the descriptive statistics for the realized (observed) volatilities of the interest rate Cetes and TIIE and the forecast volatility from the models. Prior to fitting the GARCH model an ARCH-effects test was conducted for the series under analysis. This was done in order to see if these types of models are appropriate for the data (Brooks: 2002). The test conducted was the ARCH-LM following the procedure of Engle (1982). These tests were conducted by using ordinary least squares regressing the logarithmic returns of the series under analysis against a constant. The ARCH-LM test is performed on the residuals of that regression. The test consists on regressing, in a second regression, the square residuals against a constant and lagged values of the same square residuals. The null hypothesis is that the errors are homoscedastic. An *F*-statistic was used in order to test the null. The test was carried out with different lags 2 to 10. All have the same qualitative results. Only the cases for 2 lags are reported. According to the results both series under study have ARCH effects. Under the null of homoscedasticity in the errors the *F*-statistics were 51.2398 for the Cetes and 40.9592 for the TIIE (the critical value at 95% confidence level is 3.84 for 948 degrees of freedom). Both statistics clearly reject the null in favour of heteroscedasticity on those errors. The parsimonious specification GARCH(1,1) was chosen according to results obtained from information criteria (Akaike Information Criterion and Schwarz Criterion tests). The model parameters were positive and statistically significant at the 1% level. The sum of $\alpha_1 + b_1$ was less than one.

Diagnostic tests on the models were applied to ensure that there were no serious misspecification problems. The Autocorrelation Function as well as the BDS test were applied on the standardized residuals obtained from the forecast models. Both show that these residuals were *i.i.d*. (these results are available upon request).

Table 1 shows the descriptive statistics for the realized volatility and the volatility from the forecasting models. **Figures 1** and **2** presents the logs of both interest rate series and their respective realized volatilities for

Table 1 Descriptive statistics for the daily realized volatility of futures Cetes-91 and TIIE-28 days and volatility forecasting model[1]

Model/Series	Mean	Variance	Skewness	Kurtosis	*N*
Cetes 91-days realized volatility series	2.98×10^{-4}	1.23×10^{-6}	8.6829	98.7007	951
TIIE 28-days realized volatility series	2.14×10^{-4}	4.42×10^{-7}	7.1039	72.8411	951
GARCH(1,1) model for the Cetes 91-days series	3.61×10^{-4}	2.89×10^{-7}	3.1260	16.4922	951
GARCH(1,1) model for the TIIE 28-days series	2.05×10^{-4}	4.97×10^{-8}	1.4696	5.3384	951

[1]This table reports the descriptive statistics of the realized volatility and the volatility forecasting models for the daily Cetes-91 day and TIIE-28 day futures returns. The sample size is 951 daily observations from 1st January 2003 to 29th September 2006. *N* = Number of observations.

the time frame under analysis. The daily realized volatility is defined as the log-return squared. As it can be observed in **Table 1** the four moments of the distribution of the Cetes series are the ones with higher values (the realized volatilities and the volatility forecasts). The distributions from both series are highly skewed and leptokurtic indicating non-normality of the returns and the forecast estimates.

Figure 1 Log futures Cetes 28-days and its realized volatility (left axis corresponds to the realized volatilities)

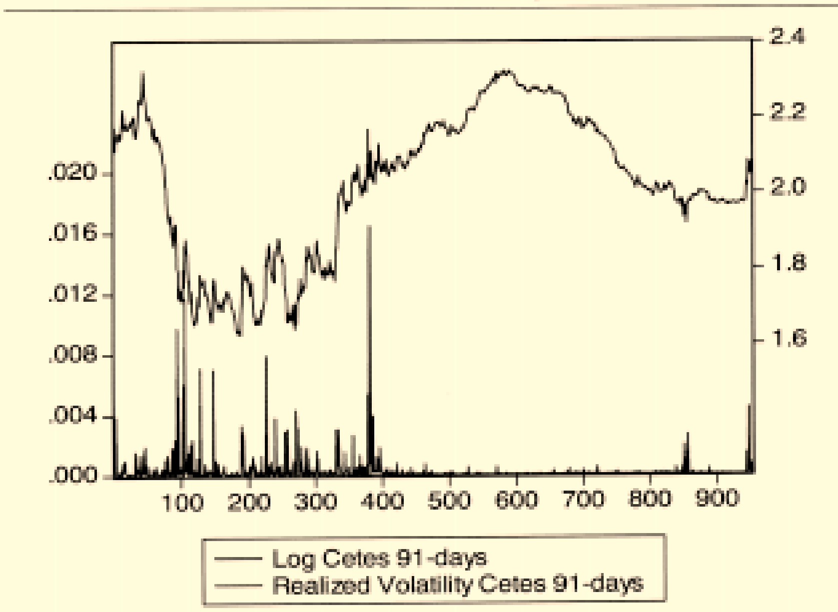
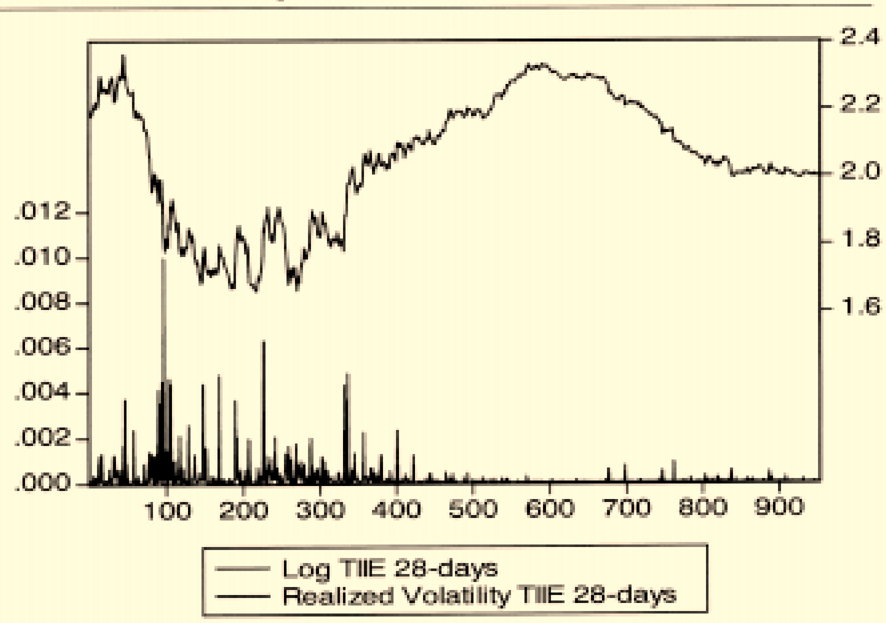


Figure 2 Log futures TIIE 28-days and its realized volatility (left axis corresponds to the realized volatilities)



7. RESULTS

7.1 Parametric method

Once the next-day volatility estimate is obtained the 95% confidence intervals are created by multiplying 1.645 by the forecasted conditional standard deviation (from the GARCH model). An analysis is made about the number of times the observed interest rate spot return was above that 95% threshold. This is formally known as a violation or an exception. Again, the positive part (right tail of the distribution) is the one of most interest given that it is positive interest rate what it causes more concern to relatively high interest rate economies thus, the interest on predicting it. Although for some economies it may be of interest the significant decreases in interest rates. For that case it is important to see the negative side of the distribution (left tail of the distribution). This is equivalent to taking a long position on the portfolio. **Figure 3** and **4** shows the spot interest rate returns and the futures confidence intervals. It can be observed that the Cetes interest rate spot returns were mostly within the 95% confidence level for the daily forecasts. However, there were violations in 25 days, which represent 2.62% of the total number of observations. Considering that a 95% confidence level is applied the model it should not exceed the VaR more than 5% (Jorion: 2001). The null hypothesis in this case is not to reject the model because it has fewer than 5% violations. The situation is qualitatively the same when TIIE series are used to calculate the 95% confidence intervals. **Figure 4** shows the same interest rate spot returns but with confidence intervals constructed with the TIIE interest rate. For this case the number of violations is 30, which represents 3.15% of the total number of observations. Again, the model is not rejected. Applying the Kupiec test as explained by Jorion (2000), the non-rejection region (interpolating) is $11 < x < 47$. So, the model is not rejected for both series under study.

7.2 Bootstrapping simulations

The methodology to carry out the simulations was explained in Section V.2 above. With the simulations it is possible to estimate Real-World Densities simulated with an ARCH model (for more information about Real-World Densities estimated with ARCH model simulations the reader can refer to Taylor (2005)). These are basically predictive densities

Figure 3 Cetes 91-days interest rate futures returns and 95% confidence levels for the one-trading day horizon VaR constructed with a GARCH(1,1) model

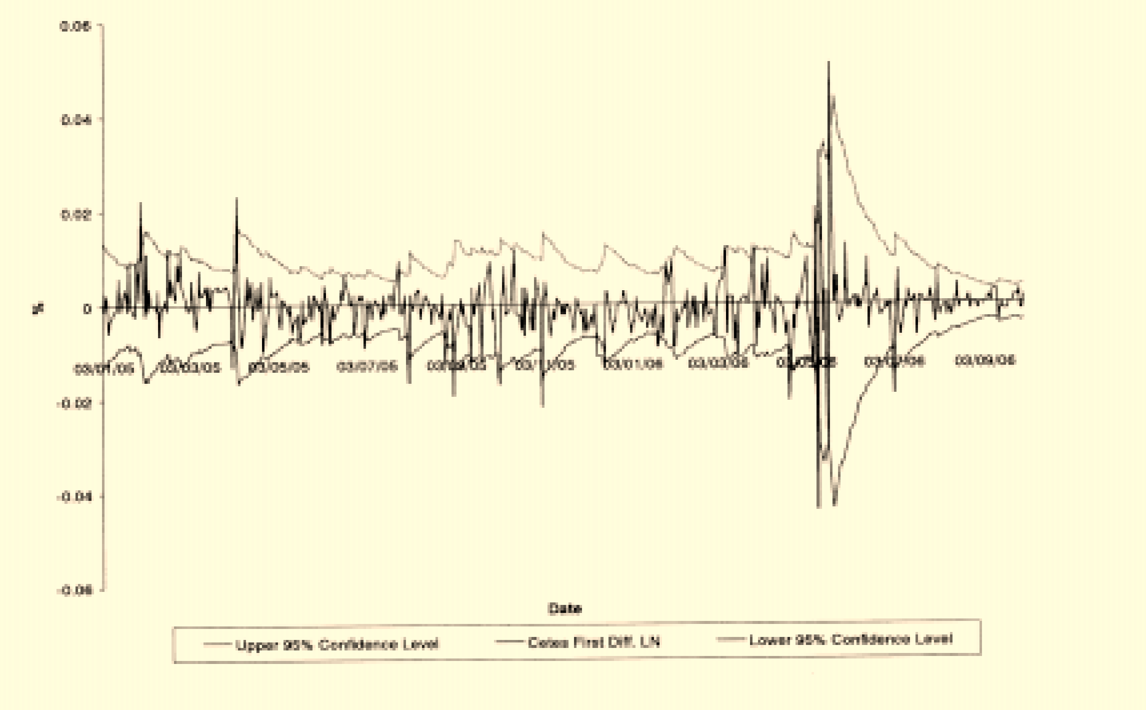
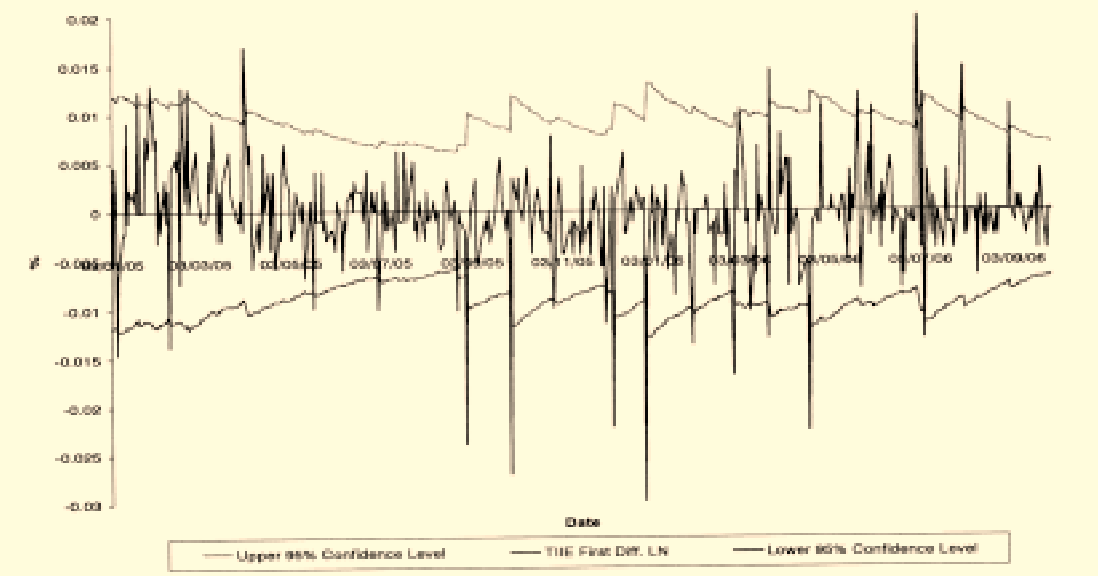


Figure 4 TIIE 28-days interest rate futures returns and 95% confidence levels for the one-trading day horizon VaR constructed with a GARCH(1,1) model

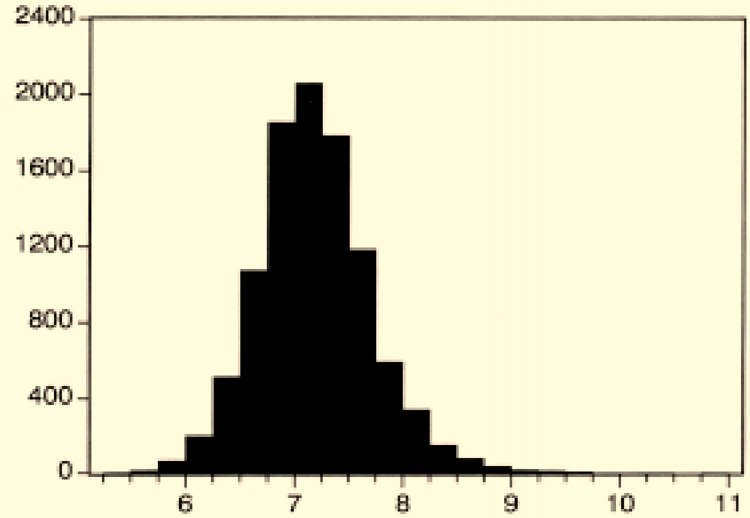


estimated in a given day for a specific date in the future. **Tables 3** and **4** show the histograms and **Figures 5** and **6** present the Real-World densities for the Cetes and TIIE series. Simulations were carried out for 29/09/06 and the Jump-off period was 18/09/06. It can be observed in the figures that the Cetes curve shows the higher maximum value and higher skewness and kurtosis. As shown with the real data the Cetes series is considerably more volatile than the TIIE series (see **Table 1**). This is also consistent with the information given in the above mention histograms (**Tables 3** and **4**). The high volatility observed in the Cetes futures is also reflected with high volatility persistence in ARCH simulations. As the time horizon increases so the confidence intervals calculated with the simulations. This can be observed in **Figure 7**. In that graph there is no event of violation or exception. This is synonymous of overestimated VaRs. The upper and lower bounds are higher compared with those for one trading day. Having a model that shows no exceptions could be costly for some portfolio investors, especially for banks. This is because unnecessary amounts of capital must be set aside in order to meet MCRRs. This is an opportunity cost of capital.

Table 4 presents the VaR for the bootstrap simulations performed for the Cetes and TIIE series respectively. The numbers of *n*-days ahead considered in the simulations were 10, 30, 90 and 180 trading days. The simulations were carried out applying the GARCH(1,1) model.

Considering the fact that the interest rate returns show autocorrelation it is necessary to do the bootstrap adjusting for an autocorrelated process (I am thankful to Alejandro Díaz de León and Daniel Chiquiar for pointing this out. I also want to thank Arnulfo Rodríguez for his assistance in helping me to incorporate the Politis and Romano (1994) methodology in the Eviews computer code). The procedure postulated by Politis and Romano (1994) is applied here. This is basically a method in which the autocorrelated returns are grouped in to non-overlapping blocks. For this case the size of these blocks is fixed during the estimation (it is also possible to have different size blocks, which vary randomly. For a more detailed explanation please refer to Politis and Romano (1994)). With the bootstrap the blocks are resample. During the simulation of the interest rate the GARCH simulated residuals (plus the original estimated parameters) are taken from the

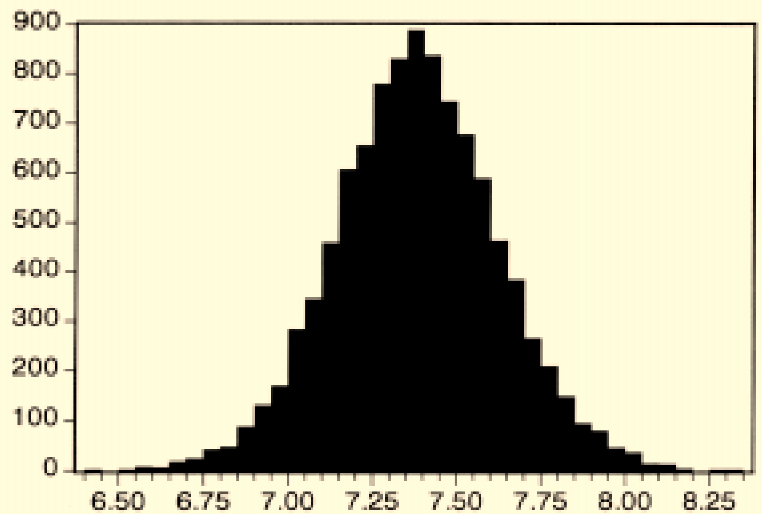
Table 2 Descriptive statistics of bootstrapping simulations using a GARCH(1,1) model for Cetes 91-day for 29/09/06. Jump-off period 18/09/06



Series: Cetes 91-day	
Sample 1 10000	
Observations 10000	
Mean	7.183355
Median	7.148057
Maximum	10.96676
Minimum	5.351201
Std. Dev.	0.526860
Skewness	0.537947
Kurtosis	4.503849
Jarque-Bera	1424.628
Probability	0.000000

This table presents the descriptive statistics for the bootstrapping simulations about Cetes 91-days. The estimation was carried out with data up to 18/09/06 (in-sample data). The simulations were made for the next 10 trading days i.e. to 29/09/06 (out-of-sample). The sample size is 951 daily observations from 1st January 2003 to 29th September 2006.

Table 3 Descriptive statistics of bootstrapping simulations using a GARCH(1,1) model for TIIE 28-day for 29/09/06. Jump-off period 18/09/06



Series: TIIE 28-day	
Sample 1 10000	
Observations 10000	
Mean	7.380724
Median	7.377539
Maximum	8.325457
Minimum	6.424968
Std. Dev.	0.241830
Skewness	0.029190
Kurtosis	3.230081
Jarque-Bera	23.47730
Probability	0.000008

This table presents the descriptive statistics for the bootstrapping simulations about TIIE 28-days. The estimation was carried out with data up to 18/09/06 (in-sample data). The simulations were made for the next 10 trading days i.e. to 29/09/06 (out-of-sample). The sample size is 951 daily observations from 1st January 2003 to 29th September 2006.

Table 4 VaR for the interest rate futures obtained with bootstrapping simulations (Estimating Real-World Densities)

Cetes 91-day	VaR *t*-day horizon	Minimum capital risk requirement *long* position	Minimum capital risk requirement *short* position
	1	5.81%	6.11%
	10	17.66%	21.41%
	30	28.95%	40.84%
	90	63.53%	158.91%
	180	90.19%	1238.80%
TIIE 28-day	1	2.33%	2.32%
	10	5.36%	5.20%
	30	9.71%	9.07%
	90	17.37%	15.42%
	180	22.99%	24.08%

This table presents the results of the bootstrap simulations (Real-World densities) adjusted for autocorrelation of the interest rate futures returns. 10,000 replications were applied to simulate the interest rate path. The time horizons considered are 1, 10, 30, 90 and 180 trading days. The Real-World densities are estimated with a GARCH(1,1) model. The sample size is 951 daily observations from 1st January 2003 to 29th September 2006.

resample blocks. The intuition is that if the autocorrelations are negligible for a length greater than the fixed size of the block, then this moving block bootstrap will estimate samples with approximately the same autocorrelation structure as the original series (Brownstone and Kazimi: 1998). Thus, with this procedure the autocorrelated process of the residuals is almost replicated and it is possible to obtain a more accurate simulated interest rate series.

From **Table 4** it can be observed that for one trading day short positions (short positions given in fourth column) the MCRRs are about 6.11% and 2.32% for the Cetes and TIIE series respectively. The interpretation of these figures is that we can be 95% certain that we will lose no more than 6.11% for Cetes or 2.32% for TIIE of portfolio value for the next trading day. As the number of the trading days increases so the VaR time horizon. In other words, for ten trading days we will be 95% certain that we will lose no more than 21.41% for Cetes or 5.20% for TIIE of portfolio value for the next ten trading days. It is important to point out that the fact that Cetes show higher variance, skewness and kurtosis (see descriptive statistics in **Tables 1** and **2**) is reflected with higher VaR estimated and consequently with higher MCRRs. As the time horizon is increased the VaR estimates increase considerably. For the case of the Cetes series the MCRRs for 180 trading days goes as far as 1238.80%. This does contrast with the MCRRs for the TIIE series that for the same time horizon the MCRRs is only about 24.08%. The fact that ARCH-type models tend to overestimate the VaR because of volatility persistence is evident. As it can be observed for both series in **Table 4** the MCRRs quickly increase to high levels as the time horizon increases for some relatively few days. For the case of the Cetes series the MCRRs increase is even higher. As explained before the explanation to this phenomenon is related with Cetes having significantly higher values for the higher moments than those for the TIIE series. The evidence here suggests rejection of the null hypothesis that ARCH-type models do not overestimate VaR. In this sense, these results are consistent with Brooks, Clare and Persand (2000). In portfolio analysis the overestimation is considered costly. This is because unnecessary quantities of capital are set aside to meet MCRRs, which in this case are unnecessary high.

Figure 5 Bootstrapping simulations with a GARCH(1,1) model in order to obtain a Real-World Density for Cetes 91-day futures ten days ahead. Simulations for 29/09/06. Jump-off period 18/09/06

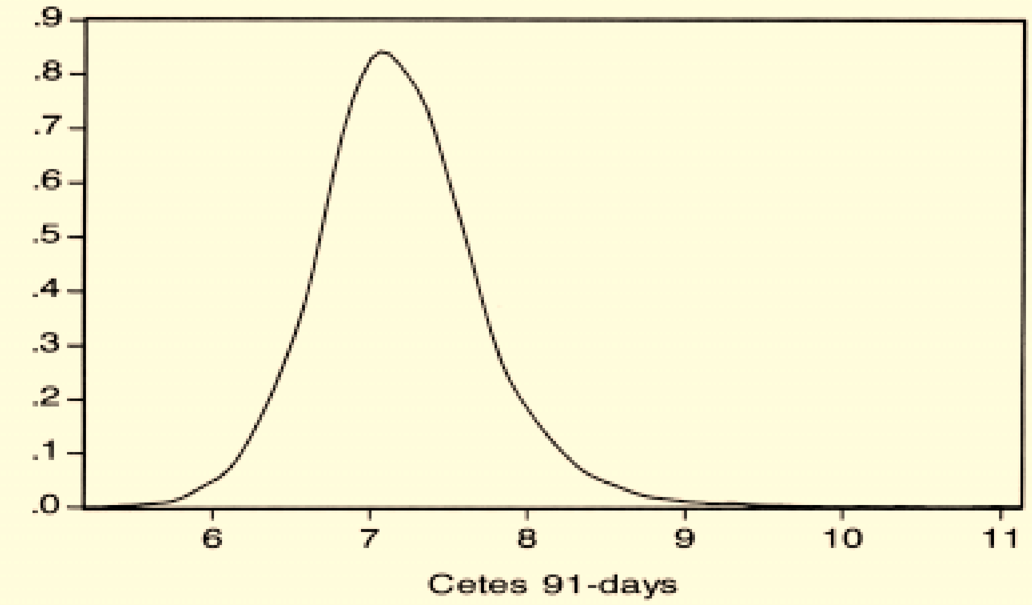


Figure 6 Bootstrapping simulations with a GARCH(1,1) model in order to obtain a Real-World Density for TIIE 28-days futures ten days ahead. Simulations are for 29/09/06. Jump-off period 18/09/06

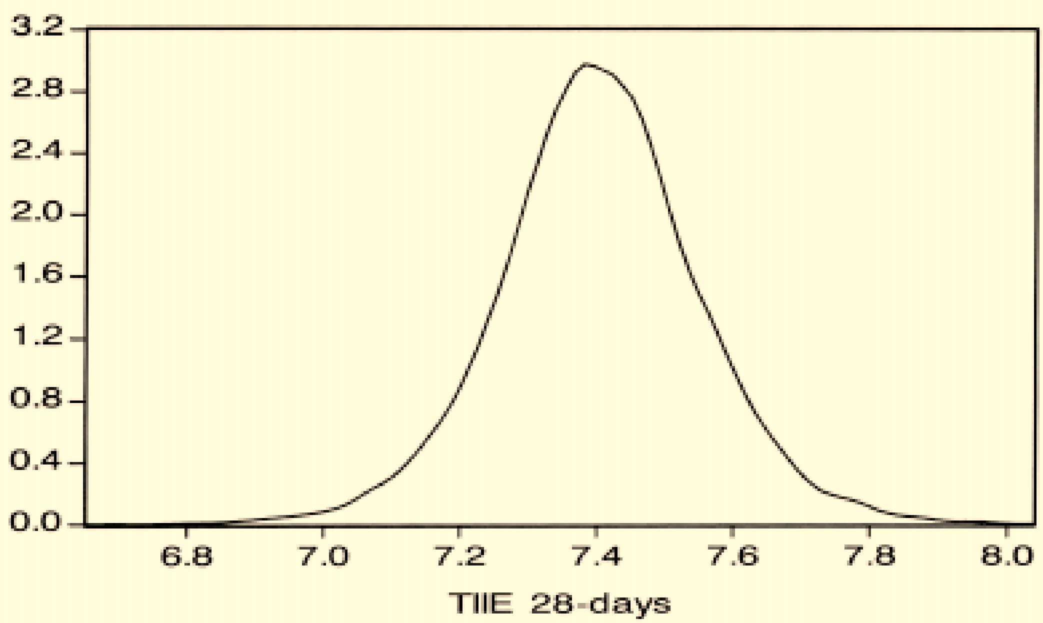
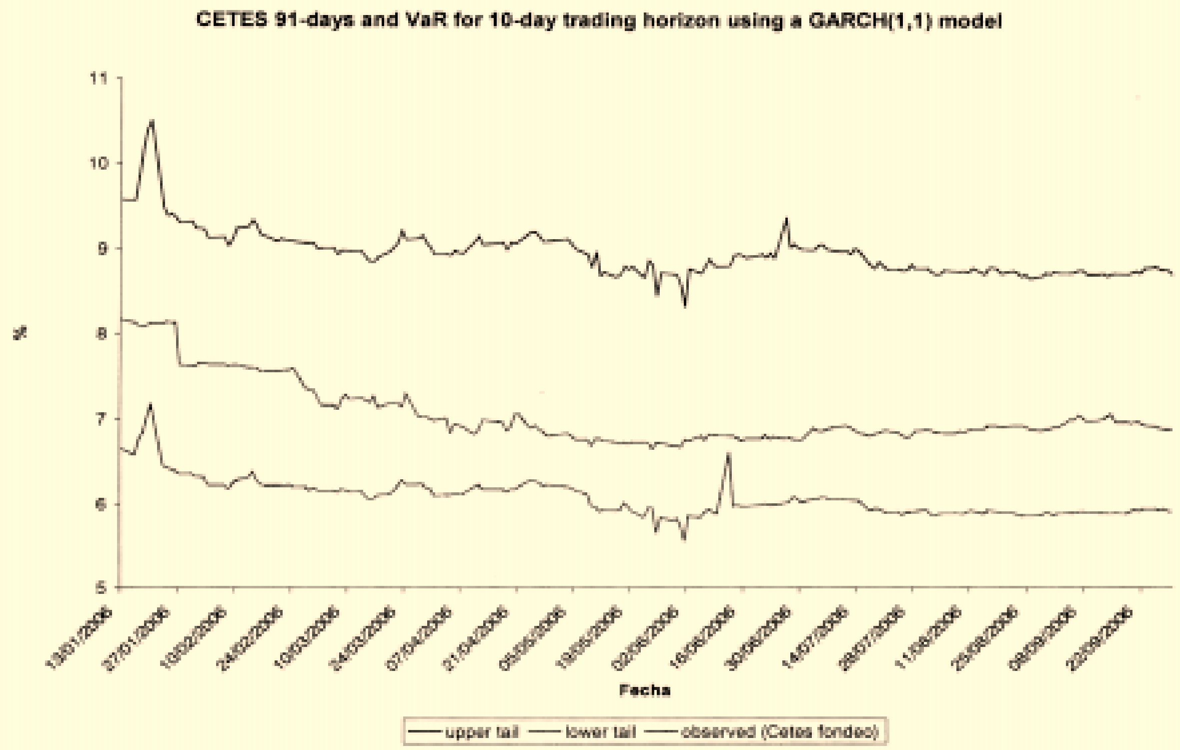


Figure 7 Cetes 91-days interest rate futures returns and 95% confidence levels for the ten-trading day horizon VaR constructed with a GARCH(1,1) model (figure in levels)



8. CONCLUSIONS

In this research project an analysis of Mexican short-term interest rate volatility was presented. The research on this project differs from that found in the literature in that interest rate futures are examined in order to draw conclusions about ARCH-type models overestimating Value at Risk (VaR). High VaR will give not optimal Minimum Capital Risk Requirements (MCRRs). This is considered costly given that investors need to set aside more capital to meet MCRRs. The results show that GARCH processes can be accurate to estimate MCRRs for one-trading day ahead time horizons. However, for time horizons of more than ten trading days the MCRRs were relatively high because of the volatility persistence captured by ARCH-type models.

In terms of forecasting short-term interest rates the estimation of the predictive Real-World densities provided confidence intervals, which can give insights about the expected range for future interest rates. In other words, it is possible to be 95% sure that the interest rate will fall within a specific confidence interval. It is recommended to extend the present research work controlling for over night volatility in the GARCH model during the estimation procedure.

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